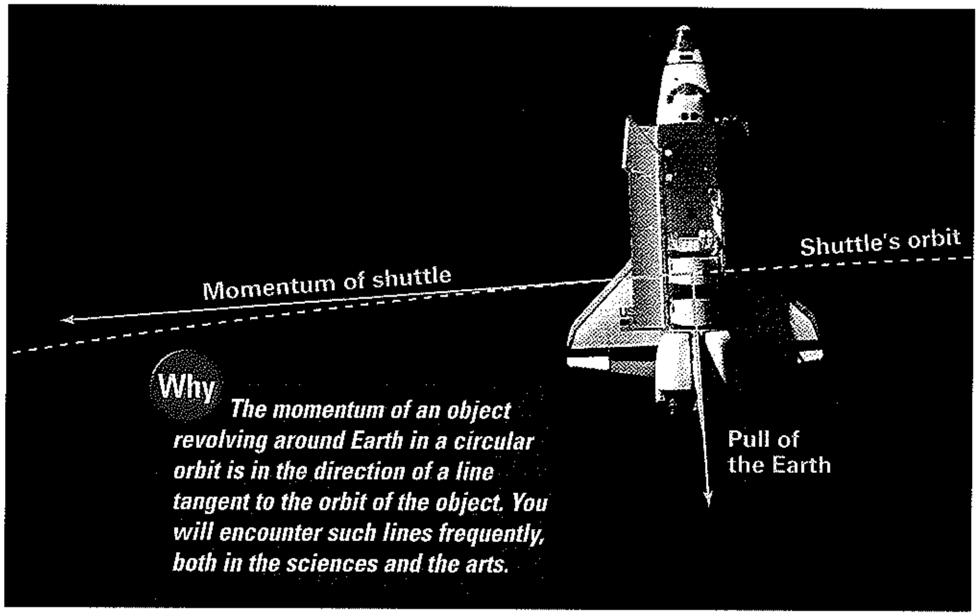
9.2

Tangents to Circles



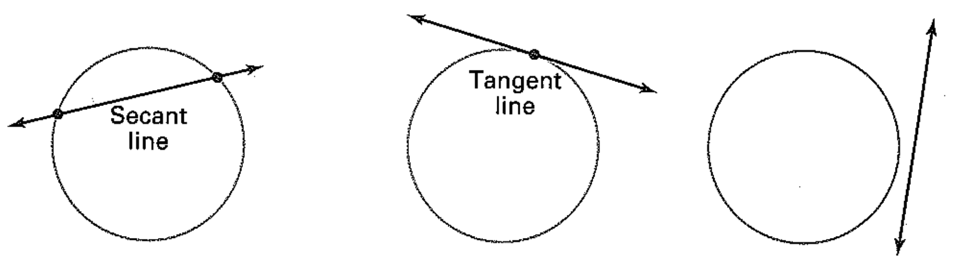
Objectives

- Define *tangents* and *secants* of circles.
- Understand the relationship between tangents and certain radii of circles.
- Understand the geometry of a radius perpendicular to a chord of a circle.

This photograph was taken by an astronaut during a space walk. The space shuttle is orbiting Earth from right to left. If it were not for the gravitational pull of the Earth, the shuttle would continue in a straight line in the direction of its momentum. It would quite literally "go off on a tangent."

Secants and Tangents

A line in the plane of a circle may or may not intersect the circle. There are three possibilities.



2 points of intersection | 1 point of intersection | 0 points of intersection

Secants and Tangents

A **secant** to a circle is a line that intersects the circle at two points. A **tangent** is a line in the plane of the circle that intersects the circle at exactly one point, which is known as the **point of tangency**. **9.2.1**

CRITICAL THINKING The word *tangent* comes from the Latin word meaning "to touch." The word *secant* comes from the Latin word meaning "to cut." Why are these words appropriate names for the lines they describe?

The Converse of the Tangent Theorem

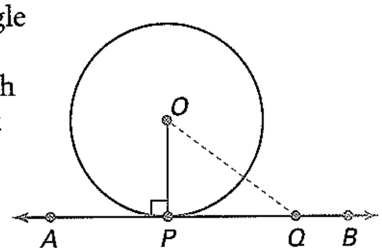
The following proof uses the definition of a circle in an interesting way. When you understand the proof, you should be able to summarize it quickly in your own words (see Exercise 18).

PARAGRAPH PROOF

Given: Point P is on $\odot O$, and $\overline{OP}$ is perpendicular to $\overleftrightarrow{AB}$.

Prove: $\overleftrightarrow{AB}$ is tangent to $\odot O$ at point P.

Proof: Choose any point on $\overleftrightarrow{AB}$ other than point P and label it Q. Draw right triangle OPQ. Since $\overline{OQ}$ is the hypotenuse of a right triangle, it is longer than $\overline{PO}$, which is a radius of the circle. Therefore, point Q does not lie on the circle. This is true for all points on $\overleftrightarrow{AB}$ except point P, so $\overleftrightarrow{AB}$ touches the circle at just one point. By definition, $\overleftrightarrow{AB}$ is tangent to $\odot O$ at point P.



Exercises

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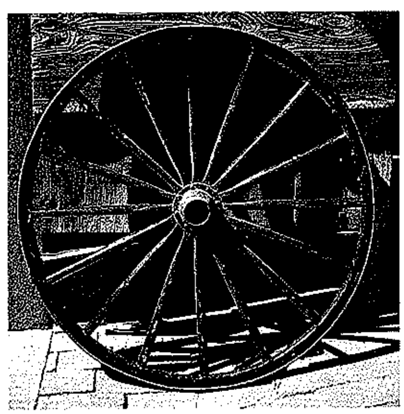
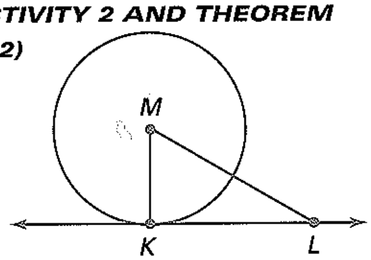
MG1 Tangents

1. Explain the three possible relationships between a line and a circle in a plane.
2. Explain how a secant intersects a circle.
3. How many lines are tangent to a circle? Explain.
4. How many lines are tangent to a circle at a given point? Explain.
5. Describe a point of tangency in the photo at right.

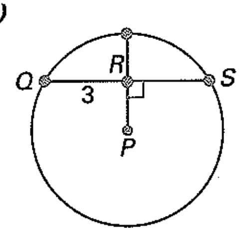


Guided Skills Practice

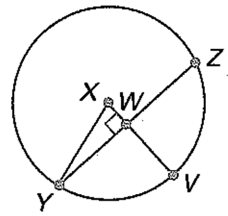
6. $\overleftrightarrow{KL}$ is tangent to $\odot M$ at K. If $KM = 1$ and $LM = 2$, find KL. ***(ACTIVITY 2 AND THEOREM 9.2.2)***



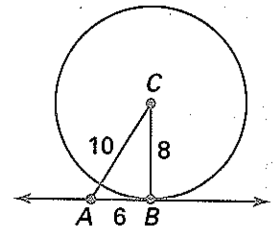
7. In $\odot P$, $QR = 3$. Find RS. ***(ACTIVITY 2 AND THEOREM 9.2.3)***



8. $\odot X$ has a radius of 13, $XW = 5$, and $\overline{XV} \perp \overline{YZ}$. Find YZ. ***(EXAMPLE AND THEOREM 9.2.3)***

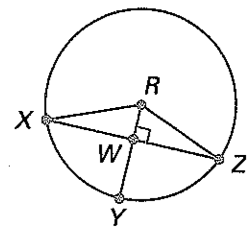


9. Verify that $\overleftrightarrow{AB}$ is tangent to $\odot C$ at B. ***(ACTIVITY 3 AND THEOREM 9.2.4)***



Practice and Apply

For Exercises 10–12, refer to $\odot R$, in which $\overline{RY} \perp \overline{XZ}$ at W.

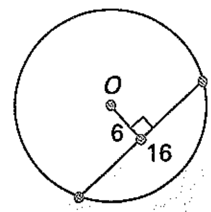


10. $\overline{XW} \cong$ __?__

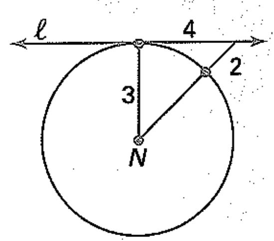
11. If $RY = 7$ and $RW = 2$, what is XW? What is WZ?

12. If $RY = 3$ and $RW = 2$, what is XW? What is WZ?

13. In the diagram at right, what is the radius of $\odot O$?



14. In $\odot N$, verify that line ℓ is a tangent by using the Converse of the Tangent Theorem.



PARAGRAPH PROOF

15. In Activity 2 you proved a theorem about a radius that is perpendicular to a chord. Write a paragraph proof of the following related theorem:

Theorem

The perpendicular bisector of a chord passes through the center of the circle. **9.2.5**

CONSTRUCTION

16. Use Theorem 9.2.5 above to construct a circle through any three noncollinear points. First draw three points not on a straight line. Label them A, B, and C. Draw $\overline{AB}$ and $\overline{BC}$. Construct the perpendicular bisector of each segment. Where is the center of the circle that contains A, B, and C? Complete the construction. How does this construction relate to Activity 2 in Lesson 1.5?

17. Use the Converse of the Tangent Theorem to construct a tangent to a circle at a given point. First draw a circle and label the center P. Choose any point on the circle and label it A. Draw $\overline{AP}$. How is the tangent line at A related to $\overline{AP}$? Complete the construction.

PARAGRAPH PROOF

18. Summarize the proof of the Converse of the Tangent Theorem from page 576 in your own words.

Algebra

Use the diagram of $\odot B$ to find the indicated lengths. Line r is tangent to $\odot B$ at T, $BT = 2$, $BS = 1$, and $WT = 5$. Round your answers to the nearest hundredth.

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MG1 Homework Help for Exercises 19-23

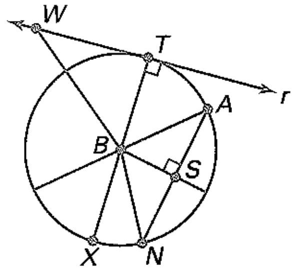
19. $BA =$ ___?___

20. $SA =$ ___?___

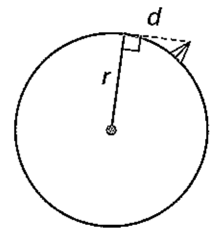
21. $SN =$ ___?___

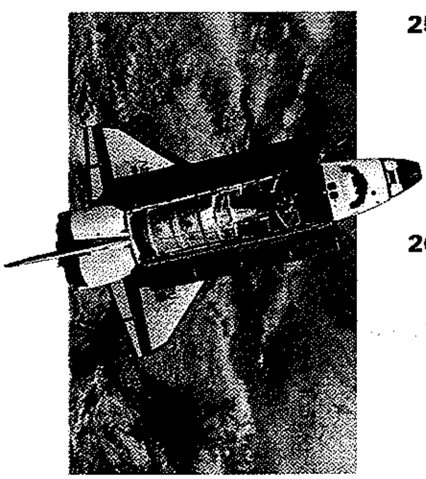
22. $BW =$ ___?___

23. $XT =$ ___?___

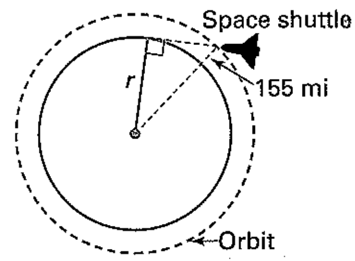


24. COMMUNICATIONS A radio station installs a VHF radio tower that stands 1500 ft tall. What is the maximum effective signal range of the tower? The diagram suggests a way to use tangents to solve the problem. Use the Pythagorean Theorem to find d. (The diameter of Earth is approximately 8000 mi.)



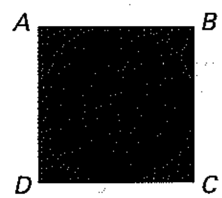


25. SPACE FLIGHT The space shuttle orbits at 155 mi above Earth. How far is it from the shuttle to the horizon? (The diameter of Earth is approximately 8000 mi.)



26. DESIGN An artist wants to draw the largest circle that will fit into a square. She uses the following method:

Draw square $ABCD$. Connect the midpoints of sides $\overline{AB}$ and $\overline{CD}$ with a segment. Connect the midpoints of sides $\overline{AD}$ and $\overline{BC}$ with another segment.



Construct the desired circle. How can you prove that this is the largest circle that will fit in the square? How can you prove that no part of the circle lies outside the square?

Look Back

Algebra

27. A triangle has a perimeter of 24 cm and an area of 24 cm^2. What are the perimeter and area of a larger similar triangle if the scale factor is $\frac{2}{1}$? ***(LESSONS 8.1 AND 8.6)***

28. A rectangle has a perimeter of 22 ft and an area of 22 ft^2. What are the perimeter and area of a larger similar rectangle if the scale factor is $\frac{8}{3}$? ***(LESSONS 8.1 AND 8.6)***

Algebra

29. A rectangular prism has dimensions $\ell = 12$ in., $w = 8$ in., and $h = 15$ in. What is the volume of a larger similar rectangular prism if the scale factor is $\frac{5}{3}$? ***(LESSONS 8.1 AND 8.6)***

APPLICATION

30. **ENGINEERING** A cylindrical water tower has a radius of 30 ft and a height of 100 ft. What is the volume of a larger similar water tower if the scale factor is $\frac{7}{5}$? ***(LESSONS 8.1 AND 8.6)***

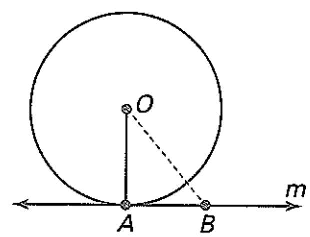
Look Beyond

CHALLENGE

PARAGRAPH PROOF

Answer the questions below to prove the Tangent Theorem.

31. Suppose that the Tangent Theorem is *false.* That is, suppose that line m is tangent to $\odot O$ at point A, but that line m is *not* perpendicular to $\overline{OA}$. If this is true, then there *is* some segment with endpoint O, different from $\overline{OA}$, that is perpendicular to line m. Call that segment $\overline{OB}$. Then $\triangle OBA$ is a right triangle. What is the hypotenuse of $\triangle OBA$? Which segment is longer, $\overline{OA}$ or $\overline{OB}$?



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MG1 Spirals

32. Point B must be in the exterior of the circle because m is a tangent line. What does this imply about the relative lengths of $\overline{OA}$ and $\overline{OB}$? Explain.

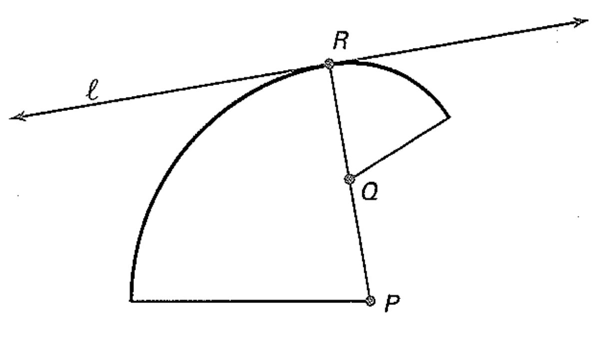
33. Compare your answers to Exercises 31 and 32. What do you observe?

34. If an assumption leads to a contradiction, it must be rejected. This is the basis for a type of proof known as an *indirect proof* or a *proof by contradiction.* Explain how the argument above leads to the desired conclusion.

PORTFOLIO ACTIVITY

CONSTRUCTING SMOOTH CURVES A curve may be made up of arcs of more than one circle. In order for arcs from two different circles to join smoothly at a point, they must have the same tangent at that point.

1. Using a compass and straightedge or geometry graphics software, draw $\odot P$ with radius $\overline{PR}$. Construct line ℓ tangent to $\odot P$ at R; that is, construct a line perpendicular to $\overline{PR}$ at R (refer to Exercise 17). For another circle to have the same tangent, ℓ, its center must be on the line $\overleftrightarrow{PR}$. Why?



2. Choose a point on $\overline{PR}$ that is not P or R and label it Q. Construct a circle centered at Q with radius $\overline{QR}$. $\odot P$ and $\odot Q$ will have the same tangent at R.

3. Try tracing part of your construction in a different color. Starting on $\odot P$ near R, trace until you get to R, and then continue tracing on $\odot Q$. The curves should join smoothly at R.

WORKING ON THE CHAPTER PROJECT

You should now be able to complete Activity 2 of the Chapter Project.