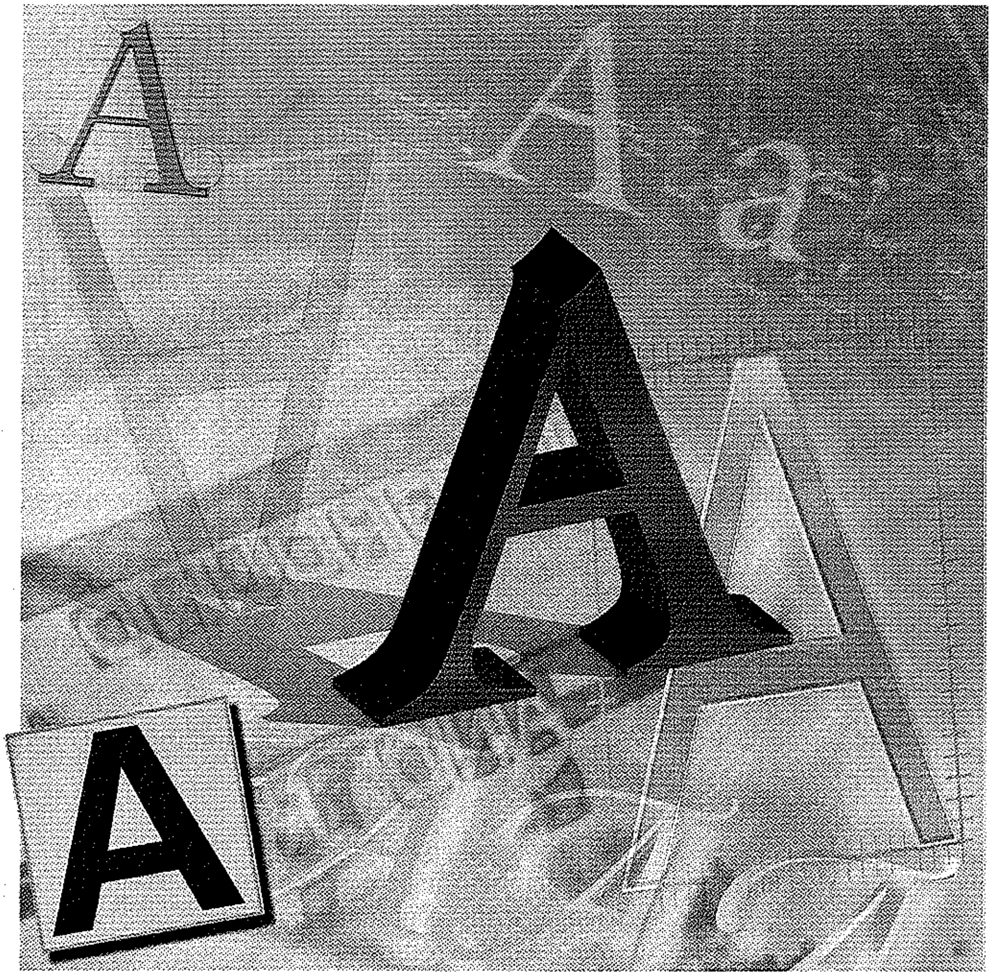
8.4

The Side-Splitting Theorem



The capital letter A may be embellished in endless ways for reasons of beauty and style. But in its most basic form, it suggests a geometry theorem.

Objectives

- Develop and prove the Side-Splitting Theorem.
- Use the Side-Splitting Theorem to solve problems.

Why *When you hear the phrase "side splitting," you may think of laughter. In geometry, it refers to a useful theorem.*

The Side-Splitting Theorem

As you will see in Example 2, people have been solving problems about proportions in triangles since ancient times. One useful result is the Side-Splitting Theorem.

Recall from the Triangle Midsegment Theorem in Lesson 4.6 that the midsegment of a triangle is parallel to one side of the triangle. The following theorem applies to any segment that is parallel to one side of a triangle.

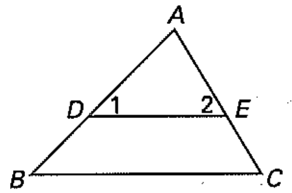
Side-Splitting Theorem

A line parallel to one side of the triangle divides the other two sides proportionally. **8.4.1**

Lo[illegible] [illegible]ing

[illegible] $\overline{BC}$

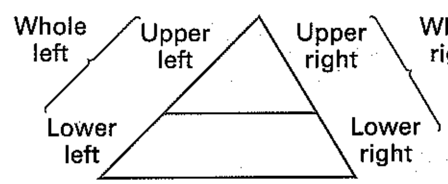
[illegible] $\frac{EC}{AE}$



[illegible]ts	Reasons
[illegible]	Given
[illegible] $m\angle 1$ [illegible] $m\angle 2$	If ‖ lines are cut by a transversal, corresponding angles are ≅.
[illegible] $\triangle ADE$	AA Similarity Postulate
[illegible] AE	Polygon Similarity Postulate
5. $AD + DB = AB$ $AE + EC = AC$	Segment Addition Postulate
6. $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$	Substitution Property
7. $\frac{AD}{AD} + \frac{DB}{AD} = \frac{AE}{AE} + \frac{EC}{AE}$	Addition of fractions
8. $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	Simplify.
9. $\frac{DB}{AD} = \frac{EC}{AE}$	Subtraction Property of Equality

CRITICAL THINKING What other proportions can you find in $\triangle ABC$ by using the Side-Splitting Theorem?

The diagram at right may help you remember the proportions in a triangle with a segment parallel to one side. The following are some of the proportions resulting from the Side-Splitting Theorem:



$$\frac{\text{upper left}}{\text{lower left}} = \frac{\text{upper right}}{\text{lower right}} \qquad \frac{\text{upper left}}{\text{upper right}} = \frac{\text{lower left}}{\text{lower right}}$$

$$\frac{\text{upper left}}{\text{whole left}} = \frac{\text{upper right}}{\text{whole right}} \qquad \frac{\text{lower left}}{\text{whole left}} = \frac{\text{lower right}}{\text{whole right}}$$

EXAMPLE 1 Use the Side-Splitting Theorem to find x in the triangle below.

SOLUTION

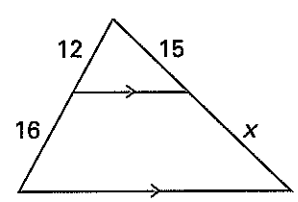
Choose a proportion that includes x as a single term.

$$\frac{\text{upper left}}{\text{lower left}} = \frac{\text{upper right}}{\text{lower right}}$$

$$\frac{12}{16} = \frac{15}{x}$$

$$12x = 240$$

$$x = 20$$



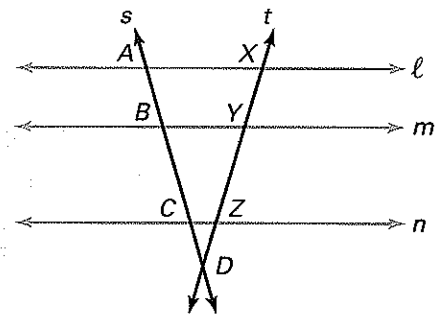
The following is a corollary of the Side-Splitting Theorem:

Two-Transversal Proportionality Corollary

Three or more parallel lines divide two intersecting transversals proportionally. **8.4.2**

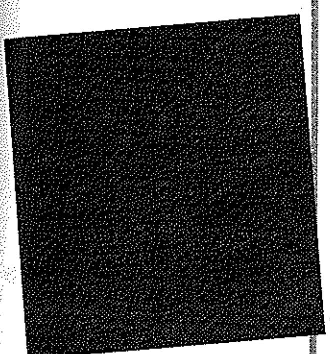
In the diagram at right, lines ℓ, m, and n are parallel, with transversals s and t.

One proportion that results from Corollary 8.4.2 is $\frac{AB}{BC} = \frac{XY}{YZ}$.

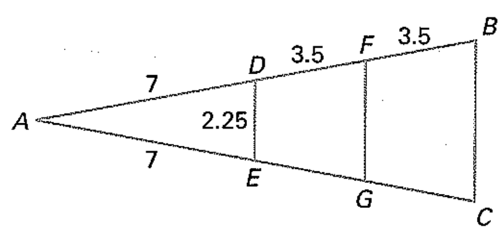


CRITICAL THINKING How can you tell that $\triangle ADX \sim \triangle BDY$?

EXAMPLE 2



CULTURAL CONNECTION: AFRICA Students in ancient Egypt studied geometry to solve practical problems involving the pyramids. This problem is based on a problem in a papyrus copied in 1650 B.C.E. by the scribe Ahmes from a source that may date back to 2000 B.C.E.



In the diagram above, $\overline{DE}$, $\overline{FG}$, and $\overline{BC}$ are parallel, $AD = AE = 7$ cubits, $DF = FB = 3.5$ cubits, and $DE = 2.25$ cubits. Find the remaining lengths.

SOLUTION

Use the Two-Transversal Proportionality Corollary:

$$\frac{AD}{DF} = \frac{AE}{EG} \quad \Rightarrow \quad \frac{7}{3.5} = \frac{7}{EG} \quad \Rightarrow \quad EG = 3.5$$

and

$$\frac{DF}{FB} = \frac{EG}{GC} \quad \Rightarrow \quad \frac{3.5}{3.5} = \frac{3.5}{GC} \quad \Rightarrow \quad GC = 3.5$$

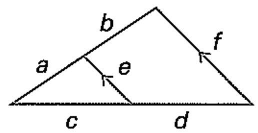
Use the definition of similar triangles:

$$\frac{AD}{AF} = \frac{DE}{FG} \quad \Rightarrow \quad \frac{7}{10.5} = \frac{2.25}{GF} \quad \Rightarrow \quad GF = 3.375$$

$$\frac{AD}{AB} = \frac{DE}{BC} \quad \Rightarrow \quad \frac{7}{14} = \frac{2.25}{BC} \quad \Rightarrow \quad BC = 4.5$$

Exercises

Communicate

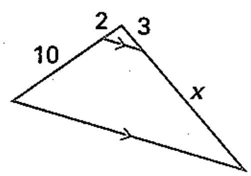


1. Use the examples on page 526 to make a list of proportions in the figure at right.
2. Are all isosceles triangles similar? Explain or give a counterexample.
3. Are all equilateral triangles similar? Explain or give a counterexample.
4. Are all right triangles similar? Explain or give a counterexample.
5. How does the capital letter A relate to the Side-Splitting Theorem? If the cross bar is horizontal, what is true of the places where it intersects the sides of the letter? Would this be true of an italic (slanted) A, as well?

Guided Skills Practice

Use the Side-Splitting Theorem to find *x*. *(EXAMPLE 1)*

6.



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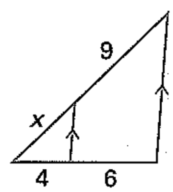

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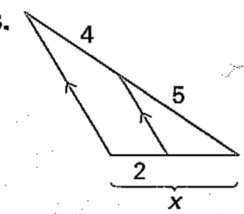
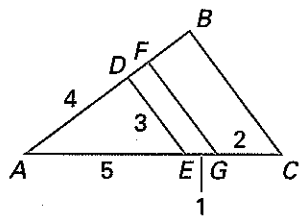


Find the indicated measurements. *(EXAMPLE 2)*

9. *DF*
10. *FB*
11. *FG*
12. *BC*



Practice and Apply

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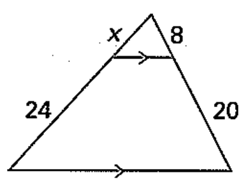
MG1 Homework Help for Exercises 13-20

In Exercises 13–20, use the Side-Splitting Theorem to find *x*. In some exercises, there may be more than one possible value for *x*.

13.



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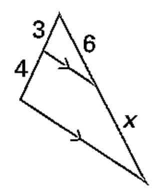

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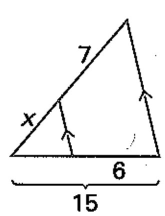

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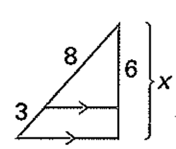
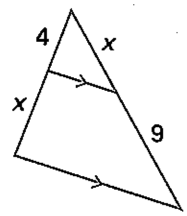


Algebra

17.



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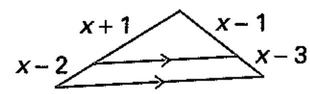

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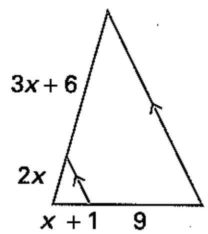

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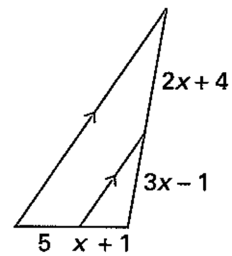
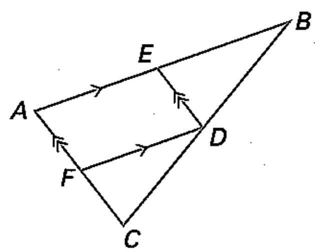


Name all similar triangles in each figure. State the postulate or theorem that justifies each similarity.

21.



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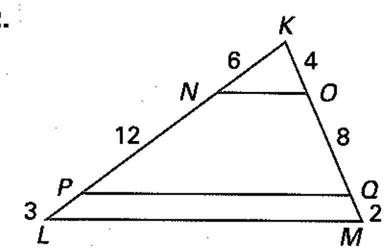
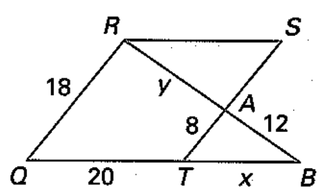


Algebra

Find *x* and *y* in each figure below.

23. *QRST* is a parallelogram.



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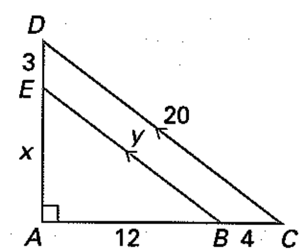

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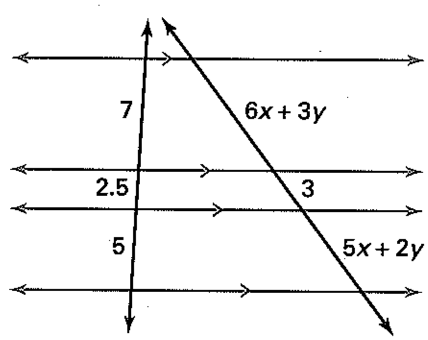
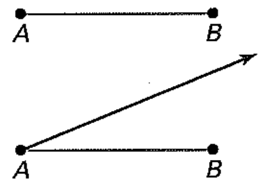
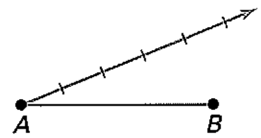


The Side-Splitting Theorem can be used to divide a segment into any number of congruent parts using a compass and straightedge.

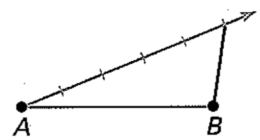
26. Draw a segment and label its endpoints A and B. Using your straightedge, draw a ray extending from point A to form an acute angle with $\overline{AB}$.



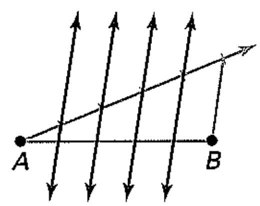
27. Set your compass to some small length (such as 1 cm) and mark off several lengths along the ray, as many as the number of parts you wish to divide the segment into.



28. Connect the last mark to point B with a segment. Construct lines parallel to this segment through each mark of the compass.



29. Explain why the parallel lines in the last figure divide $\overline{AB}$ into five congruent segments.



30. Draw a segment that is 15 cm long. Use the method described above to divide the segment into seven congruent segments.

CONNECTION

COORDINATE GEOMETRY The distance between two parallel lines is measured along a line perpendicular to both. In Exercises 31–34, you will explore the distances between parallel lines in a coordinate plane.

31. Graph the lines $y = x$, $y = x + 2$, and $y = x - 3$ in a coordinate plane. How can you verify that these lines are are parallel?

32. Graph the line $y = -x$ in the same plane. How can you verify that this line is perpendicular to the lines $y = x$, $y = x + 2$, and $y = x - 3$?

33. Give the coordinates of the points where the line $y = -x$ intersects the lines $y = x$ and $y = x + 2$. Find the distance between these parallel lines.

34. Graph the horizontal line $y = 3$ in the same plane. Use the Two-Transversal Proportionality Corollary to write a proportion, and solve it to find the distance between the lines $y = x$ and $y = x - 3$.

PROOF

35. Prove the converse of the Side-Splitting Theorem: If a segment divides two sides of a triangle proportionally, then the segment is parallel to the third side.