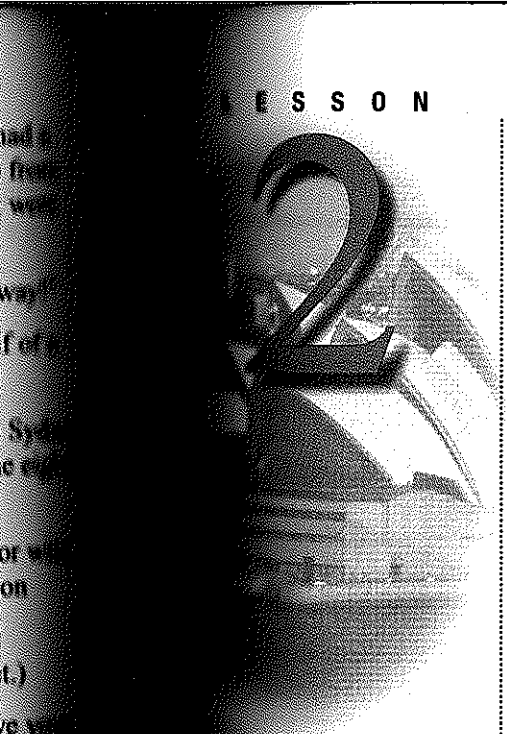
LESSON



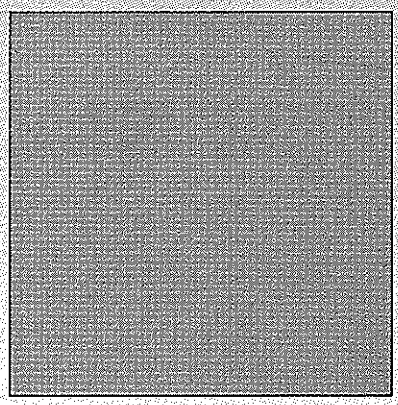
What Is a Scale Factor?

In this lesson, you will develop a precise definition of scale factor and examine how to calculate it.

Most maps and blueprints provide a scale that allows you to calculate actual distances and lengths. Depending on the map, 1 inch might represent 1 mile if the map is a detailed view of a small region, or 1 inch might represent 100 miles if the map shows a larger region. The term **scale factor** describes what reduction or enlargement from the actual size was used to obtain the map, blueprint, or picture.

Explore and Discuss

Each side of this square has length 2 inches.



(a) What do you think it means to "scale the square by a factor of $\frac{1}{2}$"?

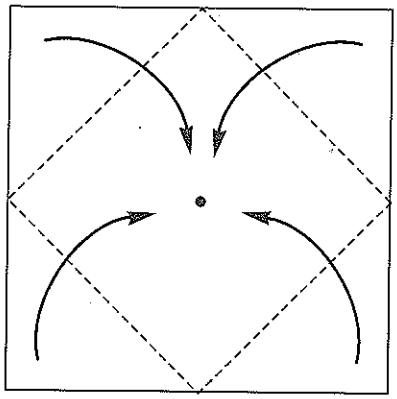
(b) Draw a figure showing what you think it would look like to scale the square by $\frac{1}{2}$. If you can think of more than one way to interpret the statement, draw a separate figure for each idea.

ACTIVITY 1 A Matter of Interpretation

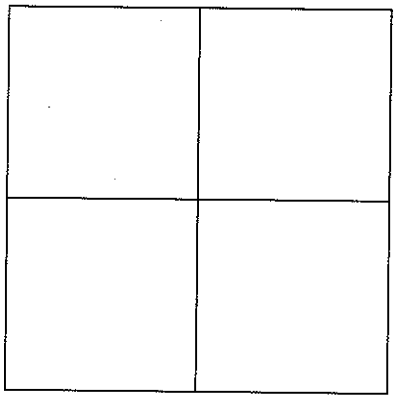
Carlo and Amy have different interpretations of what it means to scale the square by a factor of $\frac{1}{2}$. See if you agree with either of their explanations.

Why does Carlo's folding method work? How long are the sides of his new square?

Carlo: When it said to scale by $\frac{1}{2}$, I drew a square that was half the size of the first one—you know, half the *area.* Since the area of the original square is $2 \times 2 = 4$ square inches, I needed to make a square with an area of 2 square inches. One neat way to do this is to fold all four corners of the square to the center.



Amy: I thought that scaling by half meant we were supposed to draw the *sides* half as long. The first square has sides that are 2 inches long. So the scaled one should have sides that are 1 inch long. I drew a horizontal and vertical line on the square to divide the length and width in half. This gives me four squares, each scaled by a factor of $\frac{1}{2}$.



In fact, there isn't just one correct way to interpret the phrase "scale by $\frac{1}{2}$." Words can mean different things to different people. But by convention, Amy's meaning of scaling is the one that most people use.

Definition

Scaling a figure by a factor of r: When you scale a figure by a factor of r, your new figure will have lengths r times the corresponding lengths of the original figure.

The value of the scale factor, r, can be any positive number, including a fraction.

1. What features of a square are invariant when you scale it by a factor of $\frac{1}{2}$?
2. A square has a sidelength of 12 inches. How long is each side of a new square when scaling the original one by each of the following factors?

   a. $\frac{1}{4}$ **c.** $\frac{2}{3}$ **e.** 1

   b. $\frac{1}{3}$ **d.** 2 **f.** 1.3

3. If you scale a figure by the following values of r, will the new figure be smaller than, larger than, or the same size as the original one?

a. $r = \frac{3}{5}$

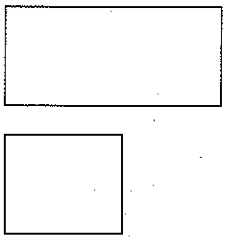
b. $r = 1$

c. $r = 3$

d. $r = 0.77$

4. Checkpoint Which of the following pairs of figures could be scaled by one half? For each pair explain why or why not.

a.



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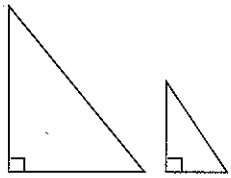

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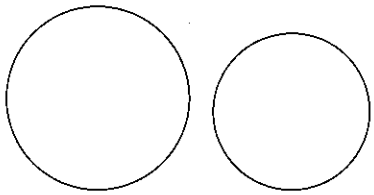
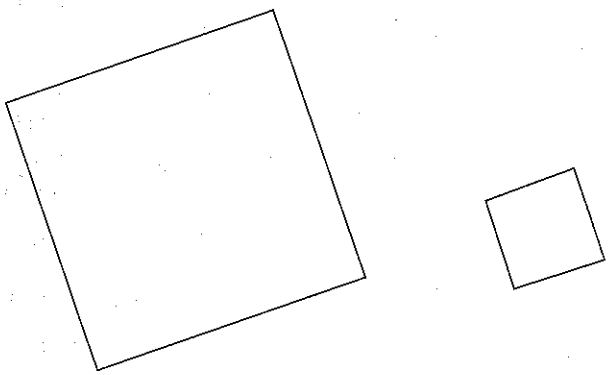


ACTIVITY 2 Calculating Scale Factors

Sometimes a scale factor isn't given but you can compare parts of figures to calculate a scale factor.

5. In each pair of pictures below, what scale factor will transform the picture on the left into the scaled picture on the right?

a.



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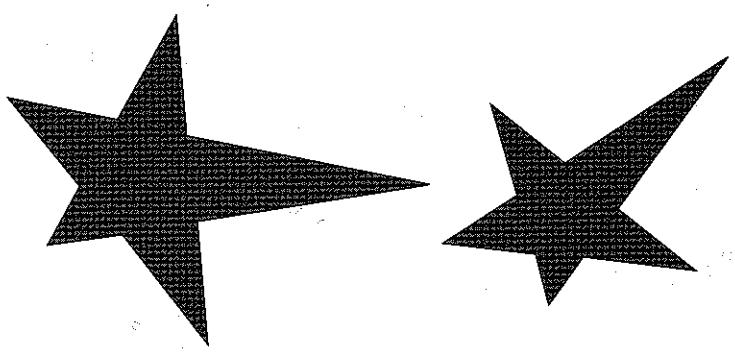
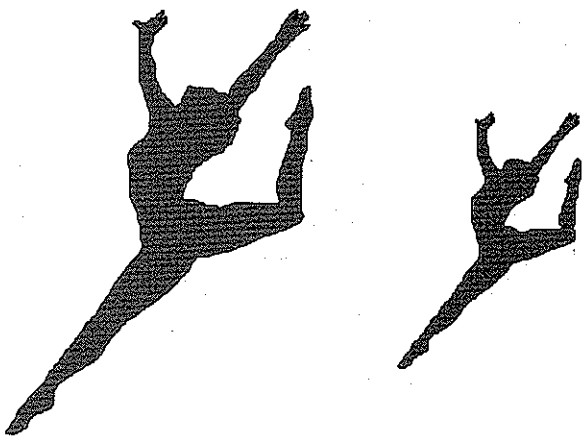


c.



d.

E E

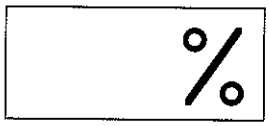
6. Suppose that the original pictures in Problem **5** are those on the right. By what factor would you scale them to get the pictures on the left?

7. Compare your answers to Problem **6** with your answers to Problem **5.** What is the relationship between them?

8. Many photocopy machines have a feature that allows you to reduce or enlarge (that is, to scale) a picture. Enter the amount 80% or some other percentage on a photocopy machine and copy a picture. (Some machines require the factor as a decimal; for this example, the amount would be 0.80.)

   a. By what factor have you scaled the picture?

   b. If you want to scale a picture by a factor of $\frac{3}{4}$, what percentage would you enter?



Copier panel for entering a reduction or enlargement percentage.

9. **Checkpoint** Label the following pairs as to whether the two scalings in the pair are the same or different.

   a. Scaling by 2 and scaling by $\frac{1}{2}$.

   b. Scaling by $\frac{1}{3}$ and scaling by 30.

   c. Scaling by $\frac{3}{5}$ and scaling by 0.6.

   d. Scaling by 1 and scaling by 100.

ACTIVITY 3 Area and Volume

These questions ask you to look at how the area or volume of a figure changes when you scale it.

10. a. Draw a square with 1-inch sides. Scale it by a factor of 2. How many copies of the 1-inch square fit inside the scaled square?

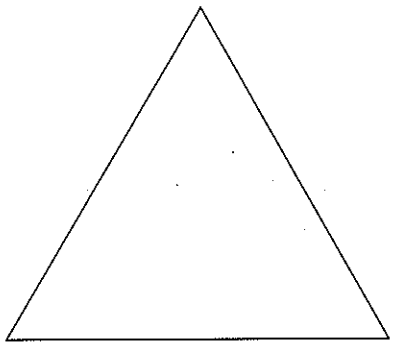
b. Start with a 1-inch square again. Scale it by a factor of 3. How many copies of the 1-inch square fit inside the scaled square?

c. If you scale a 1-inch square by a positive integer r, how many copies of it will fit inside the scaled square?

11. The equilateral triangle below has 2-inch sides.

a. Draw a scaled version of the triangle, using a factor of $\frac{1}{2}$. How many of these scaled triangles can you fit inside the original triangle?

b. Draw a scaled version of the triangle, using a factor of $\frac{1}{3}$. How many of these scaled triangles can you fit inside the original triangle?



12. A cube has edges of length 1 inch.

a. If the cube is scaled by a factor of 2, how long will the sides of the new cube be? How many copies of the original cube will fit inside the scaled cube?

b. If the original cube is scaled by a factor of 3, how long will the sides of the new cube be? How many copies of the original cube will fit inside the scaled cube?

c. If you scale the original cube by a positive integer r, how many copies of the original cube will fit inside the scaled cube?

13. Checkpoint How many 1-inch squares will fit into a 6" × 6" square that has been scaled by a factor of

a. $\frac{1}{3}$? **b.** 3? **c.** $\frac{2}{3}$?

On Your Own

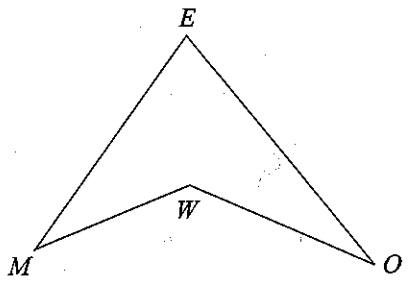
1. Give a scale factor that would change MEOW in the following ways.

a. Shrink it.

b. Enlarge it.

c. Shrink it very slightly.

d. Keep it the same size.



2. A rectangle that has width 12 inches and length 24 inches is scaled using the following factors. In each case, what are the rectangle's new dimensions?

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. 0.3

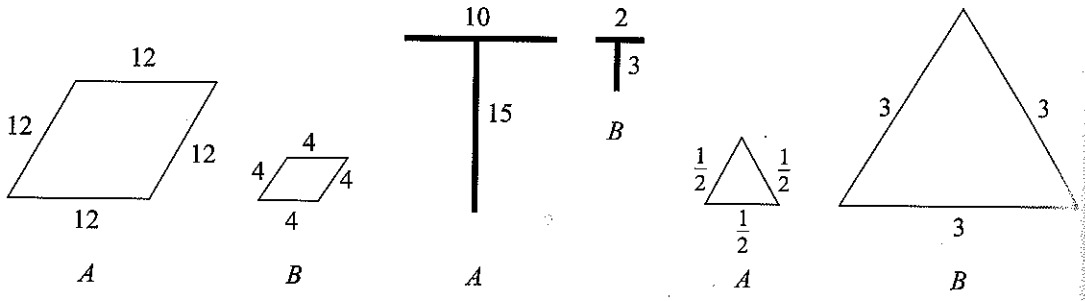
d. 2.5

e. 0.25

3. Use the figures below to answer these questions.

a. What is the scale factor that could be applied to figure *A* to get figure *B*?

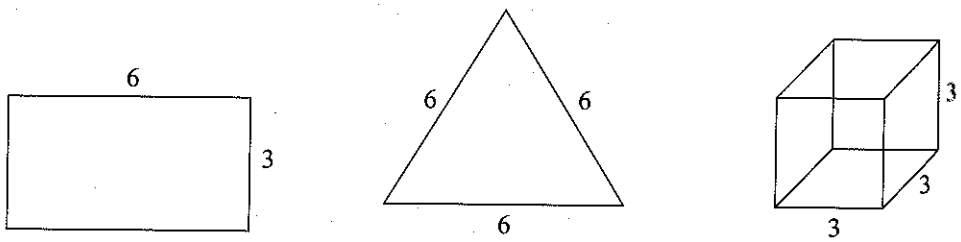
b. What is the scale factor that could be applied to figure *B* to get figure *A*?



4. Apply a scale factor of 4 to the figures below and answer these questions.

a. How long are the new sides of the figure?

b. How many of the original figures will fit into the new figure?



5. Apply a scale factor of $\frac{1}{3}$ to the figures in Problem **4.** Then answer these questions.

a. How long are the new sides of the figure?

b. How many of the original figures will fit into the new figure?

Take It Further

6. Suppose that you take a picture and scale it by a factor of $\frac{1}{2}$. You then take your *scaled* picture and scale *it* by $\frac{1}{4}$. By how much, overall, have you scaled the original picture?