Not Just for Computation:

Basic Calculators Can Advance the Process Standards

HE NATIONAL COUNCIL OF TEACHERS OF Mathematics (NCTM), the International Society for Technology in Education (ISTE), and the Conference Board of the Mathematical Sciences (CBMS) are among the many organizations that have advocated the use of various forms of technology (e.g., computer programs, graphing/scientific/ four-function calculators) to educate students. A major reason for this advocacy is captured well in the National Technology Standards comments on using technology in mathematics learning activities. "Technology can play a role in enhancing mathematical thinking, student and teacher discourse, and higherorder thinking by providing the tools for exploring and discovering mathematics" (ISTE 2000, p. 96).

These three components of the role that technology can play are embedded in the five Process Stan-



LAURA MOSS, moss@math.ohiou.edu, is an assistant professor at Ohio University, Athens, OH 45701. Her interests include the study of how instructors learn to incorporate technology into a mathematics classroom. BARBARA GRO-VER, groverbd5@verizon.net, recently retired from Ohio University, Athens, where she taught mathematics and methods classes for secondary preservice teachers. Her special interest is in the professional development of middle school,

high school, and college-level mathematics teachers.

dards (NCTM 2000) and in the Recommendations for Technology in Teacher Preparation (CBMS 2001). Graphing calculators and computer programs are often the technological tools that immediately come to mind when reading such a statement. Four-function calculators and the TI-Explorer can play the same role. (Texas Instruments has recently combined the TI-Explorer and the TI-73 graphing calculators into one device. The activities in this article relate to the TI-Explorer calculator. Although a TI-73/Explorer can be used, the keystrokes may be slightly different from those identified here.) These calculators are often more prevalent in schools around the country. This article addresses ways in which to use nongraphing calculators to enhance multiple problem-solving strategies, encourage discourse, and facilitate high-level mathematics thinking. In addition, sample activities allow students to become familiar with multiple ways of using a calculator to obtain an answer, learn to interpret the calculator display, and identify the limitations of the calculator. The two examples described in detail are purposefully simple situations. Their simplicity fosters a positive "I-can-do-this" attitude, encouraging students to contribute to discussions. The focus is on developing student abilities to think and reason. The additional examples listed are more complicated at the outset. This instructional approach can be used at many grade levels.

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The instructional recommendations are variations on one basic strategy. After students individually estimate a solution, the following work can be done in pairs or small groups before sharing with the entire class.

- Present a problem in a real-world context.
- Require students to individually record a solution estimate in ink and justify their estimate.
- Ask students to share and discuss estimates and rationales. They should do this first in pairs, before whole-class discussion. Students comment on the extent to which they are convinced of the accuracy of the thinking. The teacher remains neutral in word and body language to all contributions.
- Have students use a calculator to solve the problem, keeping track of the keystrokes they use so they can share their work with their classmates.
- Ask students to discuss the different approaches and identify similarities and differences in using the calculator.
- Interpret the solution displayed on the calculator.

We now consider two examples to illustrate this instructional strategy. The first example is a percent problem; the second example, a basic word problem, addresses a division situation in which the quotient is not a whole number.

Percent Problem

In our school, 25% of the 42 eighth graders walk to school. How many eighth-grade students walk to school? (Problem adapted from Bitter and Mikesell 1990, pp. 354–57)

At first glance, this simple, straightforward problem emphasizes a computation procedure. Students enter $.25 \times 42$, obtain an answer, and are done. However, this problem can be rich and engaging and can focus on important mathematics identified in the Process Standards, as shown in the discussion that follows.

Individually record a solution estimate in ink: Every student should have the **Percent Problem Worksheet**, pen, and pencil as well as a calculator. Each student is instructed to silently read the problem and *in ink* write an estimate of the answer along with a rationale. Using ink reduces the temptation to change an estimate and/or rationale.

As a whole class, discuss estimates and rationales: The discussion of these estimates should be conducted by displaying the procedures on an overhead projector or on the chalkboard while listening to the students' reasons. The discussion facilitator need

not comment on the accuracy of any of the estimates or rationales but should seek student opinions about their peers' contributions. Some students might estimate a solution using their knowledge of 10 percent of a number, resulting in an estimate of 8, 10, or 12. Others may use the fact that 25 percent is equivalent to 1/4 and use the fraction to obtain an estimate of 10. It is possible that some students may believe that 25 percent represents 25 students and use that as their estimate. A student could also ignore the mathematics and find an estimate based on the number of students they know walk to school. The facilitator records each of these thoughts so that the initial ideas remain public throughout the time required to work the problem. A discussion of these estimates as well as their rationales will encourage discourse, help students to develop the ability to take risks and rely on their own thinking, expose students to multiple ways of thinking about a problem, emphasize criticism of the thinking not the person, and motivate students to complete the next task (actual computation) to determine "Am I right?"

Individually use a calculator to solve: Students use their calculators to obtain a solution, noting each keystroke on their worksheets. Some students may enter 25, %, \times , 42, =. Others might enter .25, \times , 42, =. Someone may enter 42, \times , 25, %, =. Some students might enter keystrokes that yield an incorrect response.

As a whole class, discuss the different approaches to using the calculator: Questions to stimulate discussion might include these: Did everyone obtain the same result? Why? Which approach, if any, minimizes keystrokes and could be considered more efficient? By discussing different approaches and analyzing why answers are the same or different, focus students' attention on underlying mathematics concepts including decimal/percent equivalents, the meaning of various operations, and the commutative property of multiplication.

Interpret the result displayed on the calculator: Finally, the discussion should turn to the answer, 10.5. What does this number shown in the display mean? A simple interpretation is that 25 percent of 42 is 10.5. However, in the context of the original problem, how should the students interpret the value to answer the question How many students walk to school? One-half a student is not an acceptable answer; it should obviously be a whole number because we are referring to people. Thus, a discussion may now focus on interpreting this value. Should the correct answer be 10 or 11 students? Why choose one answer over the other? This discussion will allow students to bring in their personal experiences and views and may help them develop analytic as well as critical-thinking skills. Every student may contribute to this dialogue, not just those who accurately calculated the result. Both 10 and 11 could be considered correct responses as long as the student is able to adequately justify the choice.

Now let us consider a situation that involves interpreting noninteger quotients and two ways in which calculators may present those results.

Basic Word Problem

Robb wants to read 100 pages of his book before his next conference with his teacher in 7 days. How many pages should he read each day? (Problem adapted from CBMS 2001b, p. 71)

The next two steps, individually record a solution estimate in ink and discuss estimates and rationales, follow the same pattern as the percent problem (see the **Reading a Book Problem Worksheet**).

Individually record a solution estimate in ink: In this example, if an Explorer calculator is available, the teacher may instruct *all* students to use the INT \div feature or instruct some to use the INT \div feature and others to use the standard division (\div) key. Students record their keystrokes. The INT \div feature displays quotients as integer values with remainders. In this example, 100 INT \div 7 will be displayed as 14 with a remainder of 2. The standard division key displays quotients as decimals. In this example, 100 \div 7 is displayed as 14.285714 (or as many decimal places as appropriate for a particular calculator).

As a whole class, discuss the different approaches to using the calculator: As many of the readers will be aware, students are likely to enter the values of 100 and 7 in an incorrect order. A discussion of the different approaches should include a focus on the lack of a commutative property for division (e.g., 100 INT \div 7 \neq 7 INT \div 100). If some of the students used the standard division key, then additional discussion can ensue with respect to the difference in answers displayed. What does the calculator do differently in each case? How could they imitate the calculator's computation in the two situations with paper and pencil?

Interpret the result displayed on the calculator: What is the meaning of 14 r2 in the one case and 14.285714 in the other? What does the 14 represent? (He must read 14 pages per day.) What does the 2 represent? (If Robb reads exactly 14 pages per day for 7 days, he will still have 2 pages to read.) What would the students suggest Robb do to finish the assignment on time? Many "correct" answers are possible to this question (e.g., 16 pages the first day, then 14 every day thereafter; 16 pages on the last day; 15 pages on 2 of the 14 days, and 14 on the rest). The discussion should focus on reasoning, thinking, and justifying solutions. The interpretation of 14.284714 provides different challenges. The 14 means the same as it did in the 14 r2 display, but the 0.284714 means something quite different from the r2. It means 0.284714 of a page. The discussion gives students an opportunity to deepen their understanding of decimals and allows teachers to identify students who have a good understanding of decimals and those who need more help. Does it make sense for Robb to try to read 14.284714 pages per day? What are the difficulties involved? What are the advantages and disadvantages of the two formats of the answer?

On the Reading a Book Problem Worksheet, students are asked to indicate the difference between their estimate and the actual solution. This value can be tracked by the teacher as a way to assess an individual student's progress in estimation skills. The names of the top five students who *improve* their ability to estimate each month could be posted as well as the five students who have the best average difference between their estimates and the actual answers. We suggest the number 5 because it gives several students the opportunity to have his or her name on a list, not just one best student, and can be an incentive to improve performance. Teachers can adjust this number for their individual situations. Another approach would be to have individuals set goals for themselves and track their progress toward their goals.

Additional Examples

THESE TWO EXAMPLES PRESENTED SIMPLE, straightforward situations. Textbooks include similar situations that can be used to encourage deep investigation and foster communication about mathematics concepts among the students. The teacher gains significant insights into the thinking and reasoning of students, an understanding of students' proficiency or lack thereof in using a calculator appropriately, and a more thorough ongoing assessment of students' abilities. Some additional examples of real-world contexts (and their solutions) are given below.

1. Three pizzas need to be shared among 17 people. If the pizza is to be equally divided, how much pizza does each person get? (Replace the 3 pizzas with 3 dollars or 3 dozen doughnuts for alternative situations.)

3 pizzas

Using the integer division key (INT \div): The display will be 0 r3. This display is interpreted as no one (0 people) can have a whole pizza and that 3 whole pizzas remain to be shared among the 17 people. Each person's share is 3/17 of a pizza.

Using the standard division (+) key: Each person's share is displayed as 0.1764706 of a pizza. This exact decimal value is useless in this real-life problem. This situation should generate discussion about the difficulty of actually carrying out division for either of these approaches and could include what is more likely in a real-world setting. The pizzas will get cut into reasonably equal pieces, usually 8, and some people will eat more than others.

3 dollars

Using 3 divided by 17 will result in the same calculator display as for the pizzas. The INT key results in 0 r3; each person gets 0 dollars and the remaining \$3 must be shared among 17. Each person's share is 3/17 of a dollar, or 0.1764706 of a dollar. These values are difficult to imagine. Since a dollar bill cannot be sliced like a pizza, it is useful to convert the \$3 to 300 cents.

INT \div **key:** 300 INT \div 17 results in 17 r11. Each person gets 17 cents, and 11 cents are left over. Discussion ensues about what to do with the 11 cents. Again, since the remainder cannot be sliced and equally divided as can the pizza, answers are "give it to charity" or distribute funds unevenly. Equal distribution of all the money is impossible.

Standard \div **key:** Using 300 \div 17 results in 17.647059. The 17 again means that each person's share is 17 cents. The 0.647059 means each person's share in addition to the 17 cents is a bit more than half a penny (0.5 is half a penny and 0.6 > 0.5). Again, this portion cannot be distributed.

3 dozen doughnuts

The 0 r3 means that no one person can have a full dozen doughnuts; the 3 dozen must be shared among the 17 people. Each person's share is 3/17 of a dozen or 0.11764706 of a dozen. It is useful to convert the 3 dozen doughnuts to 36 doughnuts to help give meaning to these values.

INT \div **key:** 36 INT \div 17 results in 2 r2; each person's share is 2 doughnuts with 2 doughnuts left over, or 2 doughnuts and 2/17 of a doughnut. The 2 doughnuts could each be divided into 17 equal pieces, and each person will get 2 of the 1/17 pieces.

Standard ÷ **key:** 36 ÷ 17 results in 2.1176471; each person's share is 2 doughnuts and 0.1176471 of a

doughnut. This fractional part for each person is about 0.1 of a doughnut. As with the pizza, in a real-world setting two people would just eat the extra doughnuts.

2. Repeat problem 1, but divide the pizzas, dollars, or doughnuts among 15 people and then 2 people. What different mathematics comes into play?

3 pizzas for 15 people

INT ÷ **key:** 3 INT ÷ 15 = 0 r3. Each person's share is 0 whole pizzas, and the remaining 3 pizzas must be divided. Each person's share is 3/15, or 1/5, of a pizza, interpreted as "divide each pizza into 5 slices and everyone takes a slice."

Standard \div key: $3 \div 15 = 0.2$; each person's share is 0.2 of a pizza, or 1/5.

3 dollars for 15 people

INT \div **key:** 3 INT \div 15 = 0 r3; each person's share is 0 dollars and the remaining 3 dollars must be divided. Each person gets 3/15, or 1/5 of a dollar, which is 20 cents.

Standard \div **key:** $3 \div 15 = 0.2$; each person's share is 0.2 of a dollar, or 20 cents. It is not necessary to convert the \$3 to 300 cents to make sense of the amount that is each person's share, however, it is a worthwhile exercise to see that the solutions are equal.

3 dozen doughnuts for 15 people

INT ÷ **key:** 3 INT ÷ 15 = 0 r3; each person's share is 0 dozen doughnuts. The remaining 3 dozen must be divided among the 15 people. Each person's share is 3/15, or 1/5 of a dozen; 1/5 of 12 = 22/5, or 2 doughnuts and 2/5 of another doughnut.

Standard \div **key:** $3 \div 15 = 0.2$; each person's share is 0.2 of a dozen doughnuts; 0.2 of 12 = 2.4, or 2 doughnuts and 0.4 of another doughnut. It is not necessary to convert the 3 dozen to 36 doughnuts, but it is worthwhile to investigate.

3 pizzas for 2 people

INT \div **key:** 3 INT \div 2 = 1 r1; each person's share is 1 whole pizza. The remaining pizza must be divided between the 2 people. Each person's share is 1 whole pizza and 1/2 of the other pizza.

Standard \div **key:** $3 \div 2 = 1.5$; each person's share is 1 whole pizza and 0.5 of a pizza.

3 dollars for 2 people

INT \div **key:** 3 INT \div 2 = 1 r1; each person's share is 1 whole dollar. The remaining dollar must be divided by

2. Each person's share is 1 dollar and 1/2 of a dollar.

Standard \div **key:** $3 \div 2 = 1.5$; each person's share is 1 dollar and 0.5 of a dollar or a total of \$1.50. Again, it is not necessary to convert the \$3 to 300, but it shows that the solutions have equal value although the numbers (1.5 and 150) and the labels (dollars or cents) are different.

3 dozen doughnuts for 2 people

INT \div **key:** 3 INT \div 2 = 1 r1; each person's share is 1 dozen doughnuts and the remaining dozen must be divided between the 2 people. Each person's share is 1 dozen and 1/2 dozen.

Standard \div **key:** $3 \div 2 = 1.5$; each person's share is 1.5 dozen doughnuts. The conversion to 36 doughnuts is not necessary, but again it is worth investigating.

3. Ask each student, "How many seconds old are you?" given a specific ending time.

Discuss issues regarding various units of time. The class needs to decide on guidelines so that all students are using the same criteria for determining the solution. If years are used, will leap years be considered or disregarded? If months are used, will every month have 30 days, should the actual number of days of each month be used, or will every month have 4 weeks (28 days)?

Suppose a student was born June 15, 1994, at 7:15 p.m. The ending time established by the class or teacher is October 20, 2005, 1:22 p.m. Count leap years, and specify the days in a month. One approach: From June 15, 1994, 7:15 p.m. to June 15, 2005, 7:15 p.m. is 11 years, with leap years in 1996 and 2004. The number of days from June 15, 2005, 7:15 p.m. to October 19, 2005, 7:15 p.m. is 126 days. June has 15 days (from June 16 to 30); July, 31 days; August, 31; September, 30; and October, 19. The number of hours and minutes from October 19, 2005, 7:15 p.m. to October 20, 1:22 p.m. is 18 hours, 7 minutes. This student has been alive 11 years, 128 days (include the 2 leap year days), 18 hours, and 7 minutes. With 60 seconds in 1 minute, 60 minutes in 1 hour, 24 hours in 1 day, 365 days in 1 year, we have 31,536,000 seconds in 1 year, 86,400 seconds in 1 day, and 3,600 seconds in 1 hour. The number of seconds this student has been alive is

- = 346,896,000 sec. + 11,059,200 sec. + 64,800 sec. + 420 sec.
- = 358,020,420 sec.

Depending on the calculator and the age of the students, the values may be displayed in scientific notation or as a decimal with an exponent. Values may get rounded along the way. Such situations raise important issues about the limitations of some calculators and the need for understanding decimals, powers of 10, and scientific notation.

4. If you could walk a million strides, how far would you walk in miles? In kilometers? Suppose you walked 3 miles per hour (mph) and could walk 8 hours a day, how long would it take you to walk 1 million strides?

A teacher can allow the students to determine the length of their stride before starting the calculations or give the students an average measure of a stride for students at their grade level. For this solution, a stride is considered to be 18 in., or 45 cm. This activity can motivate a discussion on the advantages of the metric system.

In miles:

1,000,000 strides ≈18,000,000 in. ÷
$$\frac{63,360 \text{ in.}}{1 \text{ mi.}}$$

= 18,000,000 in. g $\frac{1 \text{ mi.}}{63,360 \text{ in.}}$ ≈ 284.09 mi.

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In kilometers:

1,000,000 strides ≈ 45,000,000 cm ÷
$$\frac{100,000 \text{ cm}}{1 \text{ km}}$$

= 45,000,000 cm g $\frac{1 \text{ km}}{100,000 \text{ cm}}$ = 450 km

In time:

$$\frac{3 \text{ mi.}}{1 \text{ hr.}} \text{ g} \frac{8 \text{ hr.}}{1 \text{ day}} = \frac{24 \text{ mi}}{1 \text{ day}}$$

$$\frac{4.83 \text{ km}}{\text{hr.}} \text{ g}\frac{8 \text{ hr.}}{\text{day}} = \frac{38.64 \text{ km}}{\text{day}}$$

$$450 \text{ km} \div \frac{38.64 \text{ km}}{\text{day}} = 450 \text{ km g} \frac{1 \text{ day}}{38.64 \text{ km}} \approx 11.65 \text{ days}$$

Why are the values in the standard system different from those in metric?

5. If the federal debt is 7.92 trillion dollars, how could you express this value in terms that an average citizen would understand? What denomination of bill would you need to use to lay a path from the earth to the moon that would use all 7.92 trillion dollars?

This problem addresses issues related to limitations of a calculator, powers of 10, scientific notation, and placement of a decimal point. The national debt value is based on the U.S. Treasury Bureau of Public Debt Web site (www.publicdebt.treas. gov/opd/opdpenny.htm), which reported the number as \$7,922,079,988,192 on September 26, 2005. The estimated population can be obtained from the U.S. Census Bureau Web site (www.census.gov/ population/www/popclockus.html). The estimated population on September 27, 2005, was 297,278,263. The teacher should go to these two Web sites to obtain accurate values close to the date of their use since the debt and the population increase daily.

Average citizen understanding: Students may suggest multiple approaches that address average citizen understanding. This solution uses the approach of how much each citizen would need to pay to erase the debt.

Most nongraphing calculators display a maximum of eight digits. Consequently, discussions need to occur that address approaches to dealing with these large numbers and the impact these choices have on the level of accuracy of the results. We suggest converting to scientific notation with three significant digits to maintain a consistent level of accuracy. The national debt is then expressed as 7.92×10^{12} . The population is expressed as 2.97×10^8 . The calculator display for $7.92 \div 2.97$ is 2.66666666. Students can mentally divide 10^{12} by 10^8 . The final result is 2.66666666 × 10^4 , or 26,666.666. Each U.S. citizen's portion of the national debt is approximately \$26,700.00.

Path from the earth to the moon: The mean distance from the earth to the moon is 384,403 kilometers. All U.S. paper currency is the same size, 15.5 cm by 6.5 cm. We assume the bills will be placed end to end. We use scientific notation again.

Distance to the moon: 3.84×10^5 km = 3.84×10^{10} cm because 1 km = 1,000 m = 100,000 cm = 10^5 cm. The length of U.S. paper currency is 15.5 cm = 1.55×10^1 cm. The number of bills needed: $(3.84 \times 10^{10}$ cm) \div $(1.55 \times 10^1$ cm/bill) = 2.48×10^9 bills. The value of each bill on the path needed to equal the national debt: $(7.92 \times 10^{12}$ dollars) \div $(2.48 \times 10^9$ bills) = 3.19×10^3 dollars/bill = \$3,190/bill. U.S. currency does not include \$3,000 bills. Thus, someone could

lay \$1,000 bills from the earth to the moon, back to the earth, and back to the moon again and still have some extra bills. Many different solutions are possible with respect to the denomination of the bill used. These two results can give students (and their parents) a better sense of the meaning of our national debt.

Conclusion

SIMPLE NONGRAPHING CALCULATORS CAN BE powerful tools to enhance students' conceptual understanding of mathematics concepts. Students have opportunities to develop (a) a broad repertoire of problem-solving strategies by observing multiple solution strategies; (b) respect for other students' abilities and ways of thinking about mathematics; (c) the ability to critically analyze mathematical ideas, not people; (d) the ability to collaborate and cooperate with diverse personalities; (e) confidence and competence in doing mathematics; and (f) the ability to persevere even when a task is difficult.

Communities of learners at two levels can be fostered: small groups and whole class. Students work in pairs or small groups establishing small learning communities with a subset of their peers, and the entire class becomes a learning community as the teacher builds a supportive and nurturing environment in which students and the teacher learn from one another.

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(See worksheets on the next pages.) \Box

Percent Problem Worksheet

NAME _____

In our school, 25% of the 42 eighth graders walk to school. How many eighth-grade students walk to school?

- **1.** Estimate the solution (in ink): ______ Explain why you selected this estimate.
- **2.** Use your calculator to solve the problem. Write down all your keystrokes and which values appear in the display each time. (You may not need to fill all the spaces.)

Keystrokes	Display

3. *After* our class discussion, complete these items. My approach to using the calculator was (circle one): Correct Incorrect

If correct, I could have been more efficient (used fewer keystrokes): Yes No Support your answer.

If incorrect, the keystrokes I could have used were these:

- **4.** Describe the meaning of the answer that the calculator displayed. Explain the meaning of the whole number part and the meaning of the remainder or the decimal in terms of students who walk to school.
- **5.** Is the final answer displayed by the calculator the answer to the question asked in the problem? Explain.
- 6. The difference between my estimate and the final solution is ______.
- **7.** One thing I learned from today's discussions is this:



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Reading a Book Problem Worksheet

NAME ____

Robb wants to read 100 pages of his book before his next conference with his teacher in 7 days. How many pages should he read each day?

- **1.** Estimate the solution (in ink):______ Explain why you selected this estimate.
- **2.** Use the INT ÷ (or the ÷) feature on your calculator to solve the problem. Write down all your keystrokes and which values appear in the display each time. (You may not need to fill all the spaces.)

Keystrokes (INT÷)	Display	

Keystrokes (÷)	Display

3. After our class discussion, complete these items. My approach to using the calculator was (circle one): Correct Incorrect

If correct, I could have been more efficient (used fewer keystrokes): Yes No Support your answer.

If incorrect, the keystrokes I could have used were these:

4. Describe what the final answer displayed in the calculator means. Explain the meaning of the whole number part and the meaning of the remainder or the decimal in terms of pages to be read.

For the INT ÷ procedure:

For the ÷ procedure:

5. Is the final answer displayed by the calculator the answer to the question asked in the problem? Explain.

For the INT ÷ procedure:

For the ÷ procedure:

- 6. The difference between my estimate and the final solution is ______.
- **7.** One thing I learned from today's discussions is this:



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